UDC 532.546+533.6.011

DYNAMICS OF THE CLOSING OF PORES AT THE SHOCK WAVE FRONT

PMM Vol. 43, No. 3, 1979, pp. 511-518

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(Received April 17, 1978)

Certain peculiarities of shock and plastic wave propagation in porous media are analyzed using the equations of state of the porous medium which take into account the plastic closing of pores and the medium viscous properties, as well as the equations of motion and conservation of mass. Effects of shock wave velocity, viscosity, yield stress, and porosity on the wave propagation properties are investigated. The condition of existence of shock waves is obtained. The range of parameter values for which complete plastic closing of pores at the wave front and in the region of partial closing of the pore space is determined. The effect of strength and viscosity properties of a porous medium on the behavior of Hugoniot curves is analyzed.

The dynamics of propagation and damping of a shock wave of not very high intensity (of the order of some tens of kilobars in the case of solid porous bodies) are to a great extent determined by the pattern of closing of voids (at the indicated pressures this is the state of plastic flow in solid bodies). In a number of experimental investigations [1—3] the considerable effect of the body viscosity properties was indicated. Propagation of low intensity waves is considerably affected by the character of the load-unload diagram which determines the additional damping mechanism (besides viscosity and thermal conductivity) in such waves.

1. Let us consider a plane stationary wave in a homogeneous medium with identical spherical pores of radius a_0. In this case all characteristics of motion depend on the single variable $\zeta = z - Dt$, where D is the wave velocity. We shall analyze the plastic wave front without going into the structure of the elastic foreshock. If the relative volume of pores is not large, the stress deviator is virtually independent of the coefficient of porosity and is constant [4, 5]. The relation between porosity and pressure at the wave front is then of the form [1]

$$P - P_0 = \rho_0 D^2 (1 - \rho_0 / \rho) \tag{1.1}$$

where P_0 is the pressure ahead of the plastic wave front in the region which contains the elastic foreshock, and ρ_0 is the density of the unperturbed medium. Density variations in an elastic wave can be neglected [1, 6].

We divide the whole body into identical spherical cells, each containing one pore so that the total mass of cells in a unit of mass is unity. The cell radius must then satisfy the condition $4\pi N \rho_m (b_0^3 - a_0^3) / 3 = 1$, where ρ_m is the density of the solid material and N is the number of cells in a unit of mass. Volume variation of an individual cell defines macroscopic variations of the porosity parameter.

In a wide range of shock wave intensity variation the width of the shock wave front Δ is considerably greater that the cell characteristic dimension b_0. The relative variation of the medium macroscopic parameters (pressure, density, etc.) over the length of an isolated element is $\sim b_0 / \Delta \ll 1$. In this approximation it is possible to consider that the cell is subjected to two independent motions, viz., as a whole, at mass velocity u, and being compressed by pressure P of the medium.

Assuming that the cell retains its spherical form under compression [1, 6], we determine the porosity parameter α as the ratio of the cell total volume to the volume of the rigid phase, i.e.

$$\alpha = b^3 / (b^3 - a^3) \tag{1.2}$$

where b and a are, respectively, the instantaneous outer and inner radii of the cell. The equation which defines the motion of material toward the pore center is

$$\rho_m \frac{dv}{dt} = \frac{\partial \sigma_r}{\partial r} + \frac{2(\sigma_r - \sigma_\theta)}{r} \tag{1.3}$$

where r is the radius measured from the pore center, ρ_m is the density of the solid material, v is the mass velocity of material toward the pore center, and σ_r and $\sigma_\theta = \sigma_\varphi$ are components of the local stress tensor. The boundary condition at the pore surface is

$$\sigma_r |_{r=a} = 0 \tag{1.4}$$

In the considered pressure range the material behaves like an incompressible one that satisfies the condition of viscoplastic flow [1]

$$\sigma_r - \sigma_\theta = Y + 2\eta\,(\partial v / \partial r - v / r) \tag{1.5}$$

where Y is the yield stress and η is the viscosity coefficient. The condition of conservation of the cell mass implies that

$$r^3 - r_0^3 = a^3 - a_0^3 \tag{1.6}$$

where r_0 defines the initial position of a point with instantaneous coordinate r relative to the pore center. When $\rho_m = \text{const}$, variation of the medium density is solely due to the variation of porosity. The relation between ρ and α is defined by the formula

$$\rho = \rho_m / \alpha \tag{1.7}$$

Solution of the stated dynamic problem (1.2) – (1.7) on plastic closing of a spherical cell [1, 6] yields the relation between porosity α and pressure P_1 on the cell surface (i.e. the quantity σ_r when $r = b$). It was used in [1] as the mean pressure in the medium. Such identification corresponds to the model which defines the behavior of porous particles immersed in a liquid phase. However, when considering porous media, it is necessary to take into account that the mean pressure in the cell (with allowance for dynamic effects) is not equal P_1.

The averaging of local pressure distribution $P_m(r, t)$ obtained from the solution of (1.3) with allowance for (1.4)–(1.7) and averaged over the cell volume yields the following result (α_0 is the initial porosity)

$$P = \frac{\rho_m \alpha_0^2}{(\alpha_0 - 1)^{2/3}} \left\{ -A(\alpha)\frac{\ddot{\alpha}}{\alpha} + B(\alpha)\frac{\dot{\alpha}^2}{6\alpha} \right\} - \frac{4\eta\dot{\alpha}}{3\alpha(\alpha - 1)} + \tag{1.8}$$

$$\frac{2Y}{3}\ln\frac{\alpha}{\alpha-1}$$

$$A(\alpha) = \frac{1}{3(\alpha-1)^{1/3}} + \frac{(\alpha-1)^{2/3}-\alpha^{2/3}}{2}$$

$$B(\alpha) = \frac{1}{\alpha^{1/3}} - \frac{1}{(\alpha-1)^{1/3}} + \frac{1}{3(\alpha-1)^{4/3}}$$

The last two terms in (1.8) define the viscoplastic properties of medium and were taken from [1], while the first terms which define the contribution of inertial effects differ from those in [1].

Owing to the irreversibility of the loading-unloading process in the case of porous media [7] the behavior of material under increasing or decreasing density must be defined by different equations. Thus formula (1.8) is applicable only to the loading stage, since under conditions of unloading the deformation of medium is of the elastic type.

2. Equation (1.1), with allowance for formula (1.7), and formula (1.8), in which it is necessary to pass to the variable ζ define the structure of the shock wave front. Pressure in the elastic foreshock which corresponds to the transition of material to the plastic state is

$$P_0 = \frac{2Y}{3}\ln\frac{\alpha_0}{\alpha_0-1} \tag{2.1}$$

By solving these equations for the function of α and introducing the dimensionless variable $\xi = \zeta / a_0$ we obtain an ordinary second order differential equation which defines the wave profile. Since the variable ξ does not explicitly appear in the obtained equation, its order can be lowered by introducing the new function $g(\alpha) = d\alpha / d\xi$. As the result, we obtain the following equation:

$$\frac{dg}{d\alpha} = \left\{B(\alpha)\frac{g}{6} + \frac{4Rk(\alpha_0-1)^{2/3}}{3(\alpha-1)} - F(\alpha,\alpha_0,k)\frac{\alpha(\alpha_0-1)^{2/3}}{g}\right\}[A(\alpha)]^{-1} \tag{2.2}$$

$$F(\alpha,\alpha_0,k) = \frac{\alpha_0-\alpha}{\alpha_0^2} + \frac{2k^2}{3}\ln\frac{\alpha_0(\alpha-1)}{\alpha(\alpha_0-1)}$$

$$k = \left(\frac{Y}{\rho_m D^2}\right)^{1/2}, \quad R = \frac{\eta}{a_0\sqrt{\rho_m Y}}$$

and the parameter R^{-1} is the analog of the Reynolds number.

The boundary conditions $g(\alpha_0) = 0$ for Eq. (2.2) are satisfied in the case of a wave propagating in the direction of the ξ-axis, when $\xi \to +\infty$.

Equation (2.2) defines nonlinear damped or periodic oscillations with the type of solution dependent on the set of parameters k, R, and α_0. Let us investigate its solution in the phase plane g, α. The equation has three singular points on the line $g = 0$. The first singularity is associated with the physical process of closure or pores at the wave front when $\alpha = 1$. The other two points define the equilibrium position $(d^2\alpha / d\xi^2 = g = 0)$ or the points of intersection of the Rayleigh lines with the curve of static compression of the material. They are obtained by solving the transcendental equation

$$F\,(\alpha,\,\alpha_0,\,k) = 0 \tag{2.3}$$

which is satisfied for $\alpha = \alpha_0$ and some α_1 contained within the limits $1 < \alpha_1 < \alpha_0$.

If the wave propagates in the positive direction of the ξ-axis, formula (2.2) defines a loading wave only in the half-plane $g \geqslant 0$. To obtain a complete picture it is, however, necessary to consider the whole region.

3. Investigation of singularity properties of (2.2) when $\alpha = \alpha_0$ by general methods of the qualitative theory of differential equtions shows that the type of the singular point is determined by the two parameters k and α_0 . If $k < k_0(\alpha_0)$, where $k_0 = [3\,(\alpha_0 - 1)\,/\,(2\alpha_0)]^{1/2}$, the singularity is a saddle, and it is possible to move from point $g = 0$, $\alpha = \alpha_0$ in the directions defined by the separatrices

$$g = (\alpha_0 - \alpha)\,/\,\Delta_\pm \tag{3.1}$$

$$\Delta_\pm = (\alpha_0 - 1)^{1/3} A\,(\alpha_0) \times \left\{-\frac{2Rk}{3} \pm \left[\frac{4R^2k^2}{9} + \left(1 - \frac{k^2}{k_0^{2}}\right)\frac{(\alpha_0 - 1)^{4/3}}{\alpha_0}\,A\,(\alpha_0)\right]^{1/2}\right\}^{-1}$$

The loading wave that propagates in the direction of increasing ξ is defined by the separatrix determined with the plus sign at the root. Integrating with respect to ξ relationships (3.1) with the plus sign (and with allowance for $g = d\alpha\,/\,d\xi$) , we obtain for the shock wave profile with $\xi \to +\infty$, the asymptotic formula

$$\alpha_0 - \alpha = c\,\exp\,(-\,\xi/\Delta_+) \tag{3.2}$$

where c is the constant of integration. The quantity Δ_+ defines the wave characteristic dimension, i.e. the length along which changes of porosity and density occur e times. As R (or viscosity coefficient η) increase, Δ_+ also increases, while the separatrix angle of inclination to the α-axis decreases and tends to zero as $R \to \infty$.

When $k \geqslant k_0$ the solutions do not generally satisfy the physical requirement that in the case of a focus or center $g \geqslant 0$, or, if the singularity is a node, the integral curves issue from the point α_0 in the direction of increasing α (the behavior of curves near a singularity is similar to that shown in Fig. 1). Hence shock waves can exist in a porous medium when $k < k_0$. The quantity k_0 determines the lowest velocity of plastic shock wave propagation in the considered porous material

$$D_{\min} = \{Y\,/\,(\rho_m k_0^{2})\}^{1/2} = \{2Y\alpha_0\,/\,[3\rho_m\,(\alpha_0 - 1)]\}^{1/2} \tag{3.3}$$

4. Let us consider the conditions of pore closure at the wave front. When $R = 0$ the input equation (2.2) is integrable

$$g^2 = \frac{2\,(\alpha_0 - 1)^{2/3}}{3A\,(\alpha)}\left\{\frac{(\alpha_0 - \alpha)^2\,(2\alpha + \alpha_0)}{2\alpha_0^{2}} - k^2 H\,(\alpha,\,\alpha_0)\right\} \tag{4.1}$$

$$H\,(\alpha, \alpha_0) = \alpha_0 - \alpha + \ln\frac{\alpha_0 - 1}{\alpha - 1} + \alpha^2 \ln\frac{\alpha_0\,(\alpha - 1)}{\alpha\,(\alpha_0 - 1)}$$

A solitary wave in variables α, ξ in the phase plane corresponds to the closed curve (4.1) symmetric about the α-axis. The multiplier in front of braces in (4.1) is positive when $\alpha > 1$ and the expression within braces is positive when $k < k_0$ and vanishes at point $\alpha = \alpha_n < \alpha_0$. This condition with Eq. (1. 1) determine the Hugoniot curve (the curve of maximum deviation of quantities from their equilibrium values) for a porous medium when $R = 0$. Its comparison with the curve of static compression (curve 3 in Fig. 2) shows the minimal porosity α_n is lower in the

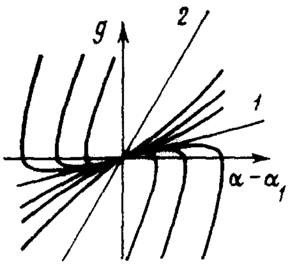

Fig. 1.

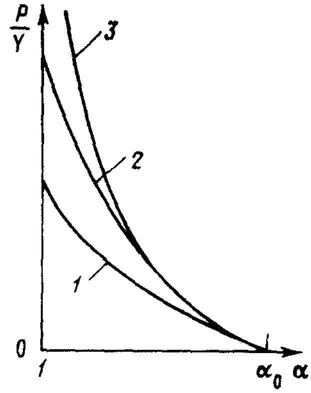

Fig. 2

dynamic case, hence the pore radius is smaller than its equilibrium size determined under conditions of static compression.

The increase of parameter R in (2. 2) (equivalent to the increase of viscosity coefficient η) with fixed k and α_0 results in increased damping the shock wave. Hence all solution in the half-plane $g \geqslant 0$ with $R > 0$ issuing from the initial point $g = 0, \alpha = \alpha_0$ lie below the trajectory with $R = 0$, and intersect the α-axis for larger values of α. This implies that when for fixed k and α_0 the trajectory with zero viscosity ($R = 0$) does not reach the value $\alpha = 1$, then no collapse of pores in the compression wave can take place ($\alpha_n > 1$) at any R.

Let us consider the solution with $R = 0$ near $\alpha = 1$. We introduce the variable $\delta = \alpha - 1$, expand the expression within the braces in (4. 1), retain in function $A(\alpha)$ the term $\sim 1 / \delta^{1/3}$ which tends to infinity, and reduce formula (4. 1) to the form

$$g^2 = \frac{\delta^{1/3}(\alpha_0 - 1)^{2/3}(\alpha_0 + 2)}{\alpha_0^2}\left\{1 - \frac{k^2}{k_c^2} + \frac{4\alpha_0^2 k^2 \delta}{(\alpha_0 - 1)^2(\alpha_0 + 2)} \ln \frac{1}{\delta}\right\} \qquad (4.2)$$

$$k_c^2 = (\alpha_0 - 1)^2 (\alpha_0 + 2) / [2\alpha_0^2 (\alpha_0 - 1 + \ln \alpha_0)]$$

Let us analyse the dependence of solution properties on parameter k. The integral curve in which k is equal k_c separates two classes of solutions. When $k > k_c$ the trajectories do not reach the singular point, forming in the neighborhood of $\delta = 0$ a saddle (Fig. 3). When $k < k_c$ the behavior of solutions is determined by the relation $g \sim \delta^{1/6}$, and the integral curves reach the singular point, becoming tangent to the g-axis at zero. The bunch of curves that have a physical meaning is bounded from above by the trajectory with $k = 0$.

The separated trajectory with $k = k_c$ reaches the singular point but at a different angle of inclination, since in this case $g \sim \delta^{2/3} \sqrt{\ln (1 / \delta)}$. This solution is also exceptional in the sense that at the reversal point the acceleration $d^2\alpha / d\xi^2 = g\,(dg / d\alpha)$ vanishes, while along all of the remaining curves it either tends to infinity when $k < k_c\,(\alpha_n = 1)$, or $(\alpha_n > 1)$ depending on number k it is equal to some quantity that tends to zero when $k \to k_c\ (k > k_c)$.

The dimensionless number k_c determines the critical velocity of the shock wave D_c. The collapse of pores can occur when the shock wave velocity exceeds the critical, i. e.

$$D > D_c = \frac{\alpha_0}{(\alpha_0 - 1)} \left\{ \frac{2Y\,(\alpha_0 - 1 + \ln \alpha_0)}{\rho_m\,(\alpha_0 + 2)} \right\}^{1/2} > D_{\min} \tag{4.3}$$

5. The behavior of solutions in the neighborhood of the singular point $g = 0,\ \alpha = 1$ with $R > 0$ and fixed α_0 and $k < k_c$ can be investigated by transforming (2.2) with allowance for $\alpha \to 1$, and introducing new variables

$$x = \frac{\alpha - 1}{(\alpha_0 - 1)\,R^3}, \quad h\,(x) = \frac{g\,(\alpha)}{(\alpha_0 - 1)\,kR^2} \tag{5.1}$$

which do not alter the singularity properties. As the result, we obtain the following equation:

$$\frac{dh}{dx} = \frac{h}{6x} + \frac{4}{x^{2/3}} + \frac{2x^{1/3}}{h} \ln \frac{M}{x} \qquad M = \frac{\exp\,[(-k_0^2 / k^2)}{\alpha_0 R^3} = \frac{\exp\,(-D^2 / D_{\min}^2)}{\alpha_0 R^3} \tag{5.2}$$

The singular points $g = 0,\ \alpha = 1$ and $g = 0,\ \alpha = \alpha_1$ in (2.2) have become $h = 0,\ x = 0$ and $h = 0;\ x = M$, respectively. Examination of a reasonably small neighborhood of the coordinate origin, which does not include the singularity at $x = M$, shows that the behavior of integral curves near the point $h = x = 0$ is similar to that shown in Fig. 3. The singular solution $h = 24x^{1/3}$ which corresponds to the critical value of parameter $R = R_1\,(k, \alpha_0)$ separates two different sets of solutions. When $R < R_1$ all solutions pass through the coordinate origin and are tangent at zero to the h-axis in conformity with the law $h \sim x^{1/6}$. The bunch of integral curves that define solutions which have a physical meaning is bounded from above by the trajectory with $R = 0$ (see formula (4.1)). For solutions of this type, including the singular solution, we have $d^2\alpha / d\xi^2 \to \infty$ as $\alpha \to 1$. Integral curves of the second set $(R > R_1)$ do not pass through the singular point and intersect the x-axis at $x > 0\ (\alpha_n > 1)$. Accelerations at reversal points are finite and tend to vanish as $R \to R_1$.

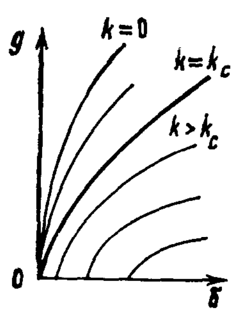

Fig. 3

The curve $R_1\,(k, \alpha_0)$ which separates the region of values of parameters R and k for which pores in the shock wave become completely closed from the region of incomplete closing of the pore space is obtained by numerical integration of equations. Functions $R_1\,(k, \alpha_0)$ are shown in Fig. 4 for several values of initial porosity; curves 1′, 2′, and 3′ correspond to $\alpha_0 = 1.5,\ 1.2,$ and 1.05, respectively. The initial points of curves on the k-axis correspond to $k_c\,(\alpha_0)$ determined by formula (4.2).

6. Let us consider the singular point α_1 whose position on the α-axis is determined by Eq. (2.3). Analysis of that equation shows that α_1 is comprised within the

limits $1 < \alpha_1 < \alpha_0$, with $\alpha_1 \to \alpha_0$ as $k \to k_0$ and $\alpha_1 \to 1$ as $k \to 0\ (D \to \infty)$.

Investigation of the properties of solutions of Eq. (2.2) near that point shows that the singularity is either of the focus or node type, depending on parameter values (it cannot be a center owing to the damping effect of viscosity). In the case of a node the separatrices are determined by the equations

$$g = \frac{(\alpha_1 - \alpha)(\alpha_0 - 1)^{2/3}}{(\alpha_1 - 1) A(\alpha_1)} \left\{ -\frac{2Rk}{3} \pm \left[\frac{4R^2k^2}{9} - \frac{(\alpha_1 - 1)}{(\alpha_0 - 1)^{2/3}} \left(\frac{2k^2}{3} - \frac{\alpha_1(\alpha_1 - 1)}{\alpha_0^2} \right) A(\alpha_1) \right]^{1/2} \right\} \quad (6.1)$$

where α_1 is obtained from (2.3). The expression under the radical vanishes for some $R = R_2(k, \alpha_0)$ which corresponds to transition from one case to another. A qualitative pattern of phase trajectories near the singular point (a node) is shown in Fig. 1 for $R \geqslant R_2$, where lines *1* and *2* are the separatrices that correspond to the plus and minus signs before the radical in (6.1).

As R increases, the angle of inclination of the first separatrix to the α -axis decreases and tends to zero as $R \to \infty$, while the angle of inclination of the second separatrix increases, tending to $\pi / 2$. When $R = R_2$ the two separatrices merge. Among the multiplicity of curves reaching the nodal point in the case of fixed parameters, only one issuing from the point $g = 0$, $\alpha = \alpha_0$ in the direction determined by formula (3.2) defines the wave front. Two situations are then possible (Fig. 1): either this curve passes through region $\alpha < \alpha_1$ and intersects the α -axis when $\alpha_n < \alpha_1$, as in the case of a focus for $R < R_2$, or it reaches the singular point from the region $g > 0$, $\alpha > \alpha_1$ $(\alpha_n = \alpha_1)$.

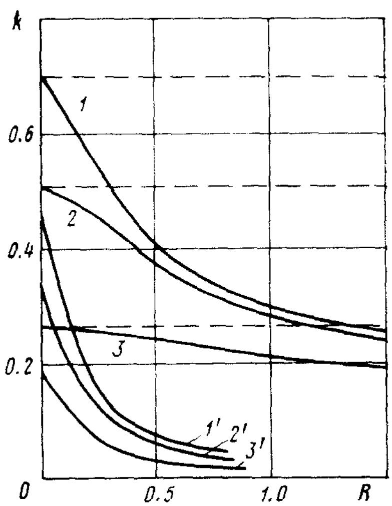

Fig. 4

Qualitatively the difference between solutions is revealed by that in the first case inertial effects predominate, while in the second it is the viscosity. In solutions of the first type the acceleration at the reversal point is nonzero. A complex oscillatory process whose definition requires the taking into account of the material elastic properties is generated in a porous medium behind the wave front. The Hugoniot curve deviates from the static compression curve of a porous material (Fig. 2) and the minimal porosity α_n in the dynamic case is lower than the equilibrium quantity α_1.

The compression wave profile in the case of the second type solutions is monotonic, with the derivatives of all quantities tending to zero as $\xi \to -\infty$. The Hugoniot curve coincides with the static compression curve, which shows that viscosity effects dominate those of inertia.

The curve of critical values of parameters $R = R_3(k, \alpha_0)$ which separates the regions that define the two solution types associated with different motions of the porous medium is determined by numerical integration of equations. Curves of function $R_3(k, \alpha_0)$ are shown in Fig. 4 for three initial porosities of the same values as for curves *1'* – *3'*. The dash lines separate regions of parameters $(k \geqslant k_0)$ in which plastic shock waves are not possible in a porous medium.

Hugoniot curves shown diagrammatically in Fig. 2 are for a porous medium and several values of parameter R. They represent a bunch of curves comprised between curve *1* corresponding to $R = 0$, and the static compression curve *3*. The typical Hugoniot curve (curve *2*) is tangent to curves *1* and *3* when R is nonzero and $\alpha = \alpha_0$. Part of this curve that corresponds to shock wave velocities which satisfy the condition $R > R_3$, coincides with the static compression curve. For considerable D, when the relationship $R_1(k, \alpha_0) < R < R_3(k, \alpha_0)$ is satisfied, the Hugoniot curve deviates from curve *3*, while for $R < R_1(k, \alpha_0)$ it merges with the axis $\alpha = 1$.

The distribution of microstresses near a pore may differ from the spherically symmetric owing to the effect of adjacent pores and to additional microscopic stresses generated by inhomogeneity (porosity) of the material. Generally, the allowance for the effect of several spherical pores complicates the problem, hence we shall restrict the evaluation of that effect to the case in which the distribution of microstresses near the pore appreciably differs from the spherically symmetric only at distances of the order of $d/2 \leqslant r \leqslant b$ (d is the average distance between pores), and without taking into consideration dynamic effects ($\alpha^{\cdot} = \alpha^{\cdot\cdot} = 0$). In this case the evaluation formula for the error $\Delta P / P$ due to the nonspherical part of stresses ΔP implies that, for instance for $\alpha = 1.2$ the relative error is of order 0.17. [9].

7. The above analysis makes possible a number of conclusions about the pattern of plastic shock wave propagation in porous media.

The lowest propagation velocity of plastic shock waves is determined by formula (3.3). The collapse of pores at the wave front does not occur if that velocity is below its critical value determined by formula (4.3). Generation of the minimal or critical velocities of shock waves depends on the strength properties of the solid medium (in a liquid with $Y = 0$ these velocities are zero).

If the medium parameters and the shock wave velocity satisfy the condition $R < R_1(k, \alpha_0)$, inertial effects predominate and the complete closure of pores takes place. When $\alpha \to 1$ the mode of pore closure in the plastic shock wave front is analogous to the law of bubble collapse in liquid, established in [10, 11]. This can be ascertained by passing from variables g and α to a and $q = da / d\xi$, where a is the radius of a pore. The law $g = c_1 (\alpha - 1)^{1/6}$ then assumes the form $q = c_2 a^{-3/2}$ (c_1 and c_2 are constants) obtained in [10, 11].

When condition $R_1(k, \alpha_0) < R < R_3(k, \alpha_0)$ is satisfied, the closure of pores is incomplete and the minimal radius of pores in the wave front is smaller than its equilibrium size. Presence of viscosity and strength of the solid material is revealed by the existence of a strong oscillatory process behind the compression wave front.

When $R > R_3(k, \alpha_0)$ the wave front structure is essentially determined by the viscoplastic properties of the medium. The Hugoniot curve merges with that of static compression of the porous medium.

The curves illustrating the critical values of parameters were obtained by analyzing the properties of singular points followed by numerical integration of equations.

REFERENCES

1. Butcher, B. M., Carroll, M. M., and Holt, A. C., Shock wave compaction of porous aluminum. J. Appl. Phys. Vol. 45, No. 9, 1974.

2. Belinskii, I. V. and Khristoforov, B. D., Viscosity of NaCl under compression shock. PMTF, No. 1, 1968.

3. Grigorian, S. S., Liakhov, G. M., and Parshukov, P. A., Spherical blast waves in earth determined by stress and strain measurements. PMTF, No. 1, 1977.

4. Green, R. J., Theory of Plasticity of Porous Materials. Mekhanika. Sb. Perev., No. 4, 1973.

5. Gurson, A. L., Continuum theory of ductile rupture by void nucleation and growth, Pt. 1. Yield criteria and flow rules for porous ductile media. Trans ASME, Ser. D. J. Basic Engng., Vol. 99, No. 1, 1977.

6. Carroll, M. M. and Holt, A. C., Static and dynamic pore collapse relation for ductile porous materials. J. Appl. Phys. Vol. 43, No. 4, Pt. 1, 1972.

7. Bhatt, J. J., Carroll, M. M., and Schatz, J. F., A spherical model calculation for volumetric response of porous rocks. Trans. ASME, J. Ser. E., J. Appl. Mech., Vol. E42, No. 2, 1975.

8. Dunin, S., Z., Sirotkin, V. K., and Surkov, V. V., On propagation of plastic waves in porous materials. Izv. Akad. Nauk SSSR, MTT, No. 3, 1978.

9. Dunin, S. Z. and Surkov, V. V., Equations of state of gas and water saturated media. Izv. Akad. Nauk SSSR, Fizika Zemli, No. 11, 1978.

10. Zel'dovich, Ia. B. and Raizer, Iu. P. Physics of Shock Waves and High-temperature Phenomena. Moscow, "Nauka", 1966.

11. Zababakhin, E. I., The collapse of bubbles in a viscous liquid. PMM. Vol. 24, No. 6, 1960.

Translated by J. J. D.